A. Mistakes, Errors, Accuracy, and Precision

Mistake and *error* have entirely different meanings in the parlance of science. A mistake is a blunder or unintentional action whose consequence is undesirable. Error, on the other hand, accounts for the range of values obtained from successive measurements of the same quantity, even though there was no mistake in any of the measurements. Moreover, error may be either *systematic* or *random*.

Systematic error

A systematic error causes a measurement to be always too large or, alternatively, always too small. An error of this type can be caused by a faulty measuring device. For example, if the intervals between the millimeter markings on a ruler are always 9/10 of the correct interval, any measurement made with that ruler will always be too small and inaccurate. Systematic error can also be caused by the consistent incorrect use of a very good measuring device.

Systematic errors influence the *accuracy* of a measurement, or, in other words, the agreement between a measured value of a quantity and its true value. In cases in which it is not possible to know the true value of a quantity, it is impossible to determine the accuracy of the measurement.

Random error

Random errors are evident when a measuring device, even a very accurate one, is used a number of times to make the same measurement. If a very large number of these measurements were made, the results could be described by the bell-shaped curve shown in Figure A.1. This graph is obtained by plotting an individual measurement value against the number of times that this value was observed. The most frequently observed values are those around the midpoint, whereas rarely occurring values of the measurement are found at either end of the curve. The average or mean value of this set of measurements corresponds to the maximum height of the curve.

The random errors that cause curves such as the one shown in Figure A.1 are closely linked to the *precision* of the measurements. When most of the measurements have values closely dispersed around the mean because of high precision, a very narrow and steep curve results. Low precision causes a widely dispersed curve with a low maximum, because the values of many of the measurements may differ significantly from the mean. A comparison of the curves that result from precise and imprecise measurements is shown in Figure A.2.

A measuring device does not need to be accurate to permit the high precision found in Figure A.2A. Because there is no necessary connection between accuracy and precision, it is possible to obtain measurements of high precision with a faulty measuring device.

Precision and standard deviation

Precision is the dispersion of, or closeness of the agreement between, successive measurements of the same quantity. The dispersion in a set of measure-

FIGURE A.1

A graph showing the distribution of experimental results when a single quantity is measured many times. The value of a measurement is plotted against the number of times that that value was found. The average or mean value of the measurement occurs at the maximum.

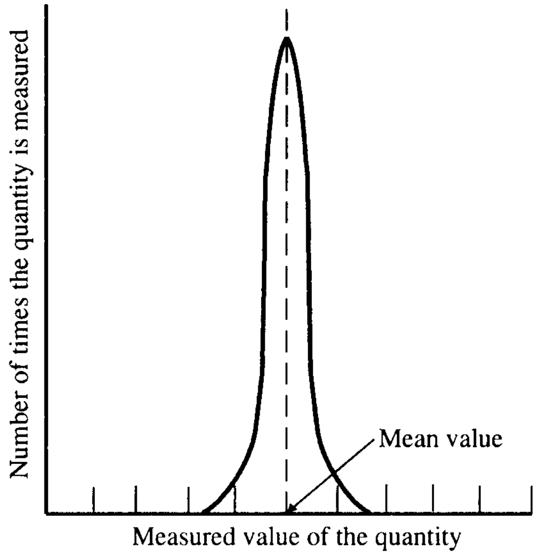
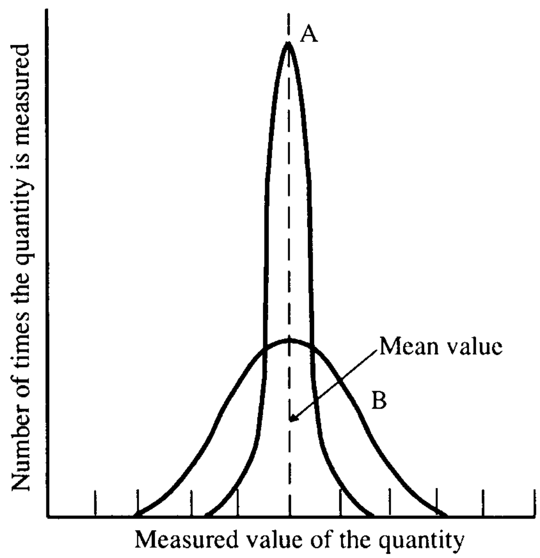


FIGURE A.2

A comparison of two distributions of experimental results. Both have the same mean value, but the measurements have high precision in A and lower precision in B.



ments is usually expressed in terms of the *standard deviation,* whose symbol is s:

$$s = \left(\frac{\Sigma\, d_i^2}{N - 1} \right)^{1/2}$$

where

Σ means "the sum of"
$d_i = x_i - \overline{x}$ = deviation
x_i = a particular value of a measurement
$\overline{x}$ = the mean value
N = the number of measurements

Measurements with high precision (Figure A.2A) are narrowly dispersed, and these measurements have a smaller standard deviation than measurements with lower precision (Figure A.2B).

Unless the number of measurements of the same quantity is very large, the calculated value of the standard deviation is only an estimate of the true standard deviation. Nevertheless, even a limited set of measurements will allow an estimate of the dispersion in the measurements to be judged.

How is the formula used? The formula states: Find the sum of the squares of the deviations, divide by one less than the total number of measurements, and take the square root of the result. The following example illustrates the use of the formula. Suppose we measure the length of an object seven times with a ruler. The values of these measurements (x_i) are 10.11 cm, 10.13 cm, 10.10 cm, 10.12 cm, 10.15 cm, 10.11 cm, and 10.12 cm. The calculation of the standard deviation of these results is shown in Table A.1.

Table A.1 An Example of Calculating a Standard Deviation

Value of Measurement x_i	Deviation $d_i = (x_i - \bar{x})$	Squared Deviation d_i^2
10.11	−0.01	0.0001
10.13	+0.01	0.0001
10.10	−0.02	0.0004
10.12	0.00	0.0000
10.15	+0.03	0.0009
10.11	−0.01	0.0001
10.12	0.00	0.0000
Sum = 70.84		Sum = 0.0016

$$Mean\ (\bar{x}) = \frac{\Sigma x_i}{N} = \frac{70.84}{7} = 10.12$$

$$s = \left(\frac{0.0016}{7-1}\right)^{1/2} = 0.016$$

You can obtain an identical result using a computer and the Internet. The Internet site is at

http://www.hmco.com/college

When a quantity, such as the length of an object, is measured several times, it is customary to report the mean value of the measurements. The dispersion or precision of the measurements can be indicated, according to one custom, by writing ± the calculated value of s after the mean. Thus we would report 10.12 ± 0.02 cm for the example in Table A.1. Note that the standard deviation of 0.016 was rounded to two figures because there are only two figures to the right of the decimal point in the mean. When we report 10.12 ± 0.02 cm as the best value for the quantity, we are stating that the length of the object probably lies between

$$10.12 + 0.02 = 10.14 \text{ cm}$$

and

$$10.12 - 0.02 = 10.10 \text{ cm}$$

Precision and significant figures

The precision of a set of measurements can also be gauged by the number of significant (that is, meaningful) figures that are used in the mean value of the measurements. This is the method that is required in the experiment on "Some Measurements of Mass and Volume."

The correct number of significant figures in a mean value will always be the number of certain digits plus one uncertain digit. In the example in Table A.1, we have shown that the length of the object probably lies between 10.10 cm and 10.14 cm, with a mean value of 10.12 cm. Clearly, the first three digits in the mean value are certain, and uncertainty occurs in the fourth digit. The precision of these measurements justifies the use of four significant figures in the mean value.

It will be instructive to consider one further example. Suppose we measure the mass of an object six times and find values of 13.34 g, 13.08 g, 13.58 g, 13.42 g, 13.29 g, and 13.45 g. As you should verify by calculations, the mean value is 13.36 g and the standard deviation is 0.17 g. The mass of the object probably lies somewhere between 13.19 g and 13.53 g. In this case, the first uncertain digit is the third digit. The precision of these measurements, therefore, will justify only three significant figures even though four digits were obtained in each measurement.

An important point should be noted here. The precision that we obtain in making a measurement is a characteristic property of the measuring device. For example, consider the ruler that was used to obtain the data in Table A.1. This device will always give a measurement in which the digit in the place immediately after the decimal point is certain, no matter what object is being measured. Moreover, the first uncertain digit will always occur in the hundredths place. As a result, the mathematical procedure for determining the precision of a measurement is required only the first time you use a measuring device. Thereafter, you should know the precision that you can obtain with that device.

Counting significant figures

We will observe the following rules when we need to count the number of significant figures in a measured quantity (Ebbing/Gammon, Section 1.5):

1. All digits are significant except zeros at the beginning of a number and possibly terminal zeros. Thus 5.46 cm, 0.546 cm, and 0.00546 cm all contain three significant figures.
2. Terminal zeros that occur to the right of a decimal point are taken to be significant. Each of the following has four significant figures: 14.10 cm, 141.0 cm, and 14.00 cm.

3. Internal zeros are significant. Thus the zero in 10.3 is significant and there are three significant figures.
4. Terminal zeros to the left of the decimal point are ambiguous and should be avoided. Ambiguity can be removed by using scientific notation.

Significant figures in calculations

Measured quantities are often used in calculations. After we complete the calculation, how many significant figures should appear in the answer? Suppose we want to determine the area of a rectangle. We measure the length and width and find these dimensions to be 115.36 cm and 3.52 cm, respectively. The area is equal to the length multiplied by the width. When we do this calculation with a pocket calculator, we obtain 406.0672 cm^2. It is incorrect, however, to use this number. The reason is simple: Its implied precision (seven significant figures) is much greater than the precision of the numbers used to obtain it.

In general, the precision of an answer to a calculation cannot exceed the precision of the measured quantities used in the calculation. We will apply two different and distinct rules to achieve this result (Ebbing/Gammon, Section 1.5):

1. When measured quantities are *multiplied* or *divided*, there should be as many significant figures in the answer as there are in the measurement with the least number of significant figures. In the calculation of the area that we just did, 3.52 cm has the least number of significant figures (three). Therefore, the answer should be reported to three significant figures, or 406 cm^2.
2. When measured quantities are *added* or *subtracted*, a different rule applies. There should be the same number of decimal places in the answer as there are in the measurement with the least number of decimal places. Suppose we wish to add 103.1 cm and 0.334 cm. Strictly speaking, the result would be 103.434 cm. But because the quantity 103.1 cm has only one decimal place whereas 0.334 cm has three, the answer is 103.4 cm.

It is important to note that any number whose value is known exactly will not affect the number of significant figures in a calculated result. For example, there are *exactly* 1×10^2 centimeters in 1 meter, so

$$1 \times 10^2 \text{ cm/m} \times 5.243 \text{ m} = 5.243 \times 10^2 \text{ cm}$$

The number of significant figures in the answer is determined by 5.243, not 1×10^2.

Rounding

Rounding is often required to obtain the correct number of significant figures. We will use the following general procedure (Ebbing/Gammon, Section 1.5). Look at the leftmost digit to be dropped.

1. If this digit is 5 or greater than 5, add 1 to the last digit to be retained and drop all digits further to the right. Thus rounding 1.2151 to three significant figures gives 1.22.

2. If this digit is less than 5, drop it and all digits further to the right. Rounding 1.2143 to three significant figures gives 1.21.

In doing a calculation of two or more steps, it is desirable to retain additional digits for intermediate answers. This ensures that small errors from rounding do not accumulate in the final result. If you use a calculator, you can simply enter numbers one after the other, performing each arithmetic operation and rounding just the final answer.